Kennedy Baptist College

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words ______________________

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Your name ______________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	48	35
Section Two: Calculator-assumed	12	12	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
8	6	
9	6	
10	9	
11	8	
12	7	
13	7	
14	9	
15	6	
16	8	
17	7	
18	9	
19	8	
S2 Total	90	
S2 Wt (×0.7222)	65%	

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Section Two: Calculator-assumed **65% (90 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(6 marks)**

A particle moves in space with position vector $\underset{\sim}{r}(t) = \begin{pmatrix} 2\cos(4t) \\ -3t \\ -2\sin(4t) \end{pmatrix}$ cm, where t is the time in seconds since its motion began.

(a) Determine the distance of the particle from its initial position after $\frac{\pi}{3}$ seconds. (3 marks)

(b) Show that the particle is moving with a constant speed. (3 marks)

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Question 9 **(6 marks)**

Particles A and B are moving with constant velocities and have initial positions $\begin{pmatrix} -8 \\ 2 \\ 10 \end{pmatrix}$ m and $\begin{pmatrix} 7 \\ 7 \\ -15 \end{pmatrix}$ m respectively. 2 seconds later A is at $\begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}$ m.

(a) Determine the velocity of A. (1 mark)

The velocity of B is $\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ m/s.

(b) Show that the paths of A and B cross, state the position vector of this point, and explain whether the particles collide. (5 marks)

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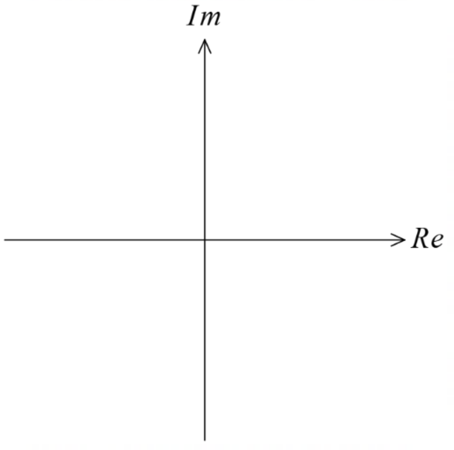
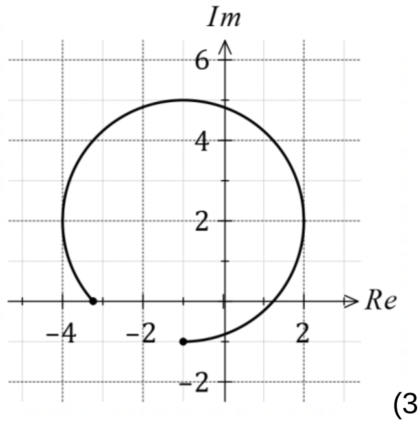
Question 10 **(9 marks)**

(a) Draw the subset of the complex plane determined by $|z + 3| > |z - 3i|$ on the axes below. (3 marks)



(b) The circular arc in the diagram represents the locus of a complex number z.

Without using $Re(z)$ or $Im(z)$, write equations or inequalities in terms of z for the indicated locus.



(3 marks)

(c) Describe the subset of the complex plane determined by $|z - 3| + |z + 3i| = 3\sqrt{2}$. (3 marks)

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Question 11 **(8 marks)**

(a) Determine the equations of all asymptotes of the graph of $y = f(x)$ when

(i) $f(x) = \dfrac{1 + 2x^2}{x(1 - 3x)}$. (2 marks)

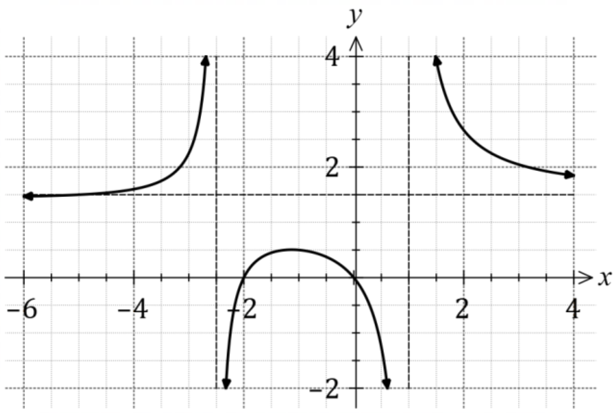
(ii) $f(x) = \dfrac{x^2 + 4}{x - 5}$. (2 marks)

(b) The graph of $y = g(x)$ is shown in the diagram, together with its three asymptotes.

The defining rule is given by

$$g(x) = \frac{ax(x + b)}{(2x + c)(x - d)}$$

where a, b, c and d are positive integer constants.



Determine, with brief reasons, the value of a, b, c and d. (4 marks)

Question 12 **(7 marks)**

Relative to an origin O located on level ground, a projectile is launched from $\begin{pmatrix} 0 \\ 7.5 \end{pmatrix}$ m with an initial velocity of $\begin{pmatrix} 20 \\ 21 \end{pmatrix}$ m/s. The motion of the projectile is only affected by a constant acceleration of $\begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$ m/s^2.

(a) Derive from the acceleration vector an expression for the position vector $\underset{\sim}{r}(t)$ of the projectile t s after its launch. (3 marks)

(b) Determine the distance travelled through the air by the projectile from when it is launched until the instant it reaches the ground, correct to the nearest 0.1 m. (4 marks)

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Question 13 **(7 marks)**

At time t seconds, $t \geq 0$, the position vector $\underset{\sim}{r}(t)$ m of a particle is given by

$$\underset{\sim}{r}(t) = \begin{pmatrix} 2e^{-0.5t} - 3 \\ 1 - 4e^{-1.5t} \end{pmatrix}$$

(a) State the position vector of the point that the particle approaches as $t \to \infty$. (1 mark)

(b) Determine the speed of the particle when $t = 4$, correct to the nearest 0.001 m/s. (3 marks)

(c) Express the Cartesian equation for the path of the particle in the form $y = f(x)$. (3 marks)

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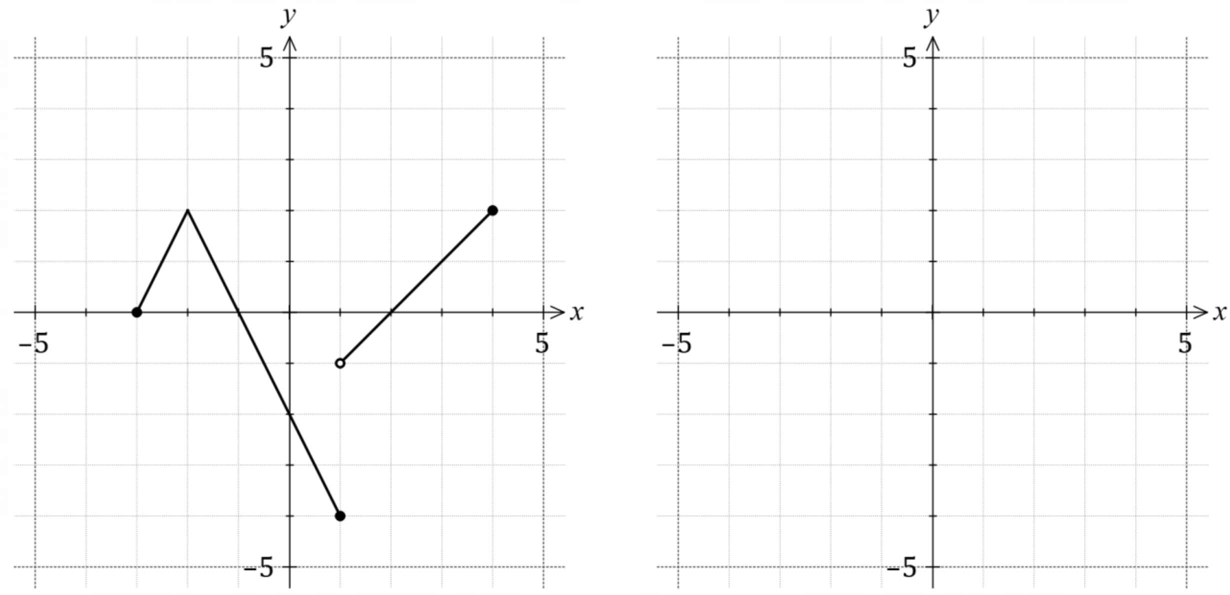
Question 14 **(9 marks)**

The graph of $y = f(x)$ is shown on the left-hand axes in the diagram below.



(a) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the right-hand axes in the diagram. (5 marks)

(b) Solve the following equations.

(i) $f(|x|) = 1.$ (1 mark)

(ii) $\left|\dfrac{1}{f(x)}\right| = 1.$ (1 mark)

(iii) $|f(x)| + f(x) = 0.$ (2 marks)

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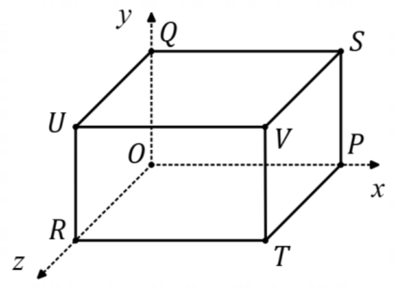
Question 15 **(6 marks)**

The diagram shows a right rectangular prism.



Relative to vertex O, vertices P, Q and R have position vectors $\begin{pmatrix} p \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ q \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ r \end{pmatrix}$.

(a) Determine vectors $\overrightarrow{PU}$ and $\overrightarrow{QT}$ in terms of p, q and r. (1 mark)

(b) Use a vector method to show that diagonals PU and QT bisect each other. (3 marks)

(c) Determine the relationship between p, q and r when $\overrightarrow{PU}$ and $\overrightarrow{QT}$ are perpendicular. (2 marks)

Question 16 **(8 marks)**

(a) Determine all solutions to the equation $z^3 - 8i = 0$ in exact polar form. (3 marks)

(b) Consider the ninth roots of unity expressed in polar form $r \text{ cis } \theta$.

(i) Determine the roots for which $0 < \theta < \frac{\pi}{2}$. (2 marks)

(ii) Use all nine roots to show that $\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right) = 0$.

(3 marks)

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Question 17 **(7 marks)**

Plane Π has equation $2x - y - 3z = 13$ and point A has coordinates $(1, 5, 4)$.

(a) Determine the coordinates of the point in Π that is closest to A. (4 marks)

Vector $\underset{\sim}{v}$ lies in plane Π, is perpendicular to the line $\dfrac{x+1}{-1} = \dfrac{y-3}{2} = \dfrac{z-1}{2}$ and $\left|\underset{\sim}{v}\right| = \sqrt{26}$.

(b) Let $\underset{\sim}{v} = a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$. Determine the value of coefficients a, b and c, given that $a > c$.

(3 marks)

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Question 18 **(9 marks)**

Let $u = \sqrt{3} + i$ and $v = \sqrt{2}\,\text{cis}\left(\frac{\pi}{30}\right)$.

(a) Determine an exact value for

(i) $\arg(uv)$. (1 mark)

(ii) $|u + i|$. (1 mark)

(b) Let $w = \frac{u^4}{v^n}$, where n is a positive integer.
Determine the minimum value of n so that w is purely imaginary. (3 marks)

The modulus of complex number z is 1 and its argument is θ, where $-\pi < \theta \leq \pi$.

(c) Determine the value of θ for which

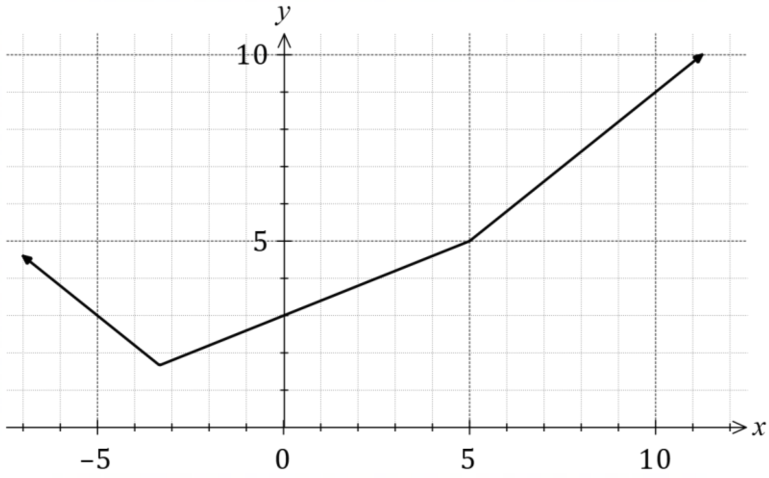
(i) $|u + z|$ is minimum. (1 mark)

(ii) $\arg(u + z)$ is maximum, where $-\pi < \arg(u + z) \leq \pi$. (3 marks)

Question 19 **(8 marks)**

Let $f(x) = |ax + b| + |cx + d|$ where a, b, c and d are constants such that $a \geq c \geq 0$.

The graph of $y = f(x)$ is shown below and passes through the points $(0, 3), (5, 5)$ and $(10, 9)$.



(a) The equation $f(x) = kx + 1$ has an infinite number of solutions. State the value of the constant k. (1 mark)

(b) Determine the value of a, b, c and d. (5 marks)

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(c) Determine the minimum value of $f(x)$. (2 marks)

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: _________